CONVECTIVE DISSIPATIVE HEAT EXCHANGE IN GENERALIZED COUETTE FLOW OF A NONLINEARLY VISCOPLASTIC LIQUID

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UDC 532.517:532.135

Dissipative heat exchange in generalized Couette flow of a nonlinearly viscoplastic liquid between two parallel plates is investigated theoretically.

We investigate the steady-state, stabilized flow of a nonlinearly viscoplastic medium between two parallel plates. The upper plate moves in its plane at the constant velocity U. A constant pressure gradient, $|\mathrm{grad}\ p| = A$, acts in the clearance. The pressure gradient may be due to a mechanical or any other factor [1,2]. The orientation of the vector $\vec{U}$ either coincides with the sense of $\vec{A}$ or is opposite to it.

It is assumed that the characteristics of the medium are independent of the temperature. The axial flow of heat is neglected. The boundary conditions are the following: I) The lower plate is isothermal ($T = T_l^* = \mathrm{const}$), while the upper, mobile plate is adiabatic; II) the upper plate is isothermal ($T = T_u^* = \mathrm{const}$), while the lower one is adiabatic.

We place the axis of abscissas Ox along the lower plate, while the axis of ordinates Oy is perpendicular to it.

As was shown in [1], depending on the rheological characteristics of the liquid, the magnitude and direction of the pressure gradient, and the plate velocity, three types of developed flow are possible: 1) flow with a quasisolid zone (core) inside the flow; 2) flow with the core adjacent to either the upper or the lower plate; 3) flow without a core in the clearance. The type of flow is determined by a pair of dimensionless parameters (α, β_0).

We shall use a generalized model to describe the rheological behavior of the liquid [3]:

$$\tau^{\frac{1}{n}} = \tau_0^{\frac{1}{n}} + (\mu_p\dot{\gamma})^{\frac{1}{m}} \tag{1}$$

with the constant rheological parameters τ_0, μ_p, m, and n.

For the initial assumptions and viscous dissipation, the heat problem is stated as follows:

$$0 = \lambda \frac{d^2T}{dy^2} + \tau \frac{dV}{dy}, \tag{2}$$

while the boundary conditions are

$$\text{I)} \quad T(0) = T_l^*, \qquad \left.\frac{dT}{dy}\right|_{y=h} = 0 \tag{3}$$

or

$$\text{II)} \quad \left.\frac{dT}{dy}\right|_{y=0} = 0, \qquad T(h) = T_u^*. \tag{4}$$

The dimensionless velocity distribution in the channel is described by the relationships [1]

$$\begin{gathered} W_1(\xi) = \frac{1}{\alpha}\sum_{k=0}^{\infty}(-1)^k\Phi_{mn}^k[(\xi_0-\xi)^{\varepsilon_k} - \xi_0^{\varepsilon_k}], \qquad 0 \leqslant \xi \leqslant \xi_1, \\ W_2(\xi) = 1 + \frac{1}{\alpha}\sum_{k=0}^{\infty}(-1)^k\Phi_{mn}^k[(\xi-\xi_0)^{\varepsilon_k} - (1-\xi_0)^{\varepsilon_k}], \quad \xi_2 \leqslant \xi \leqslant 1, \\ W_3(\xi) \equiv W_1(\xi_1) = W_2(\xi_2) = \mathrm{const}, \quad \xi_1 \leqslant \xi \leqslant \xi_2. \end{gathered} \tag{5}$$

A. V. Lykov Institute of Heat and Mass Exchange, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 7, pp. 50-58, July, 1979. Original article submitted February 2, 1978.

 0022-0841/79/3701-0790$07.50

Here, the subscript 1 pertains to the zone of shear flow at the lower plate; the subscript 2 corresponds to the shear flow zone at the upper plate; the subscript 3 refers to the zone of quasisolid flow (core);

$$\Phi_{mn}^{k}=C_{m}^{k}\frac{1}{\varepsilon_{k}}\beta_{0}^{\frac{k}{n}};\quad \varepsilon_{k}=\frac{m+n-k}{n};\quad C_{m}^{k}=\frac{m!}{k!\,(m-k)!};$$

ξ_1 and ξ_2 are the lower and the upper core boundaries, respectively; ξ_0 is the dimensionless coordinate of the plane where the shearing stress τ vanishes.

It should be noted that [1]

$$\xi_2-\xi_0=\xi_0-\xi_1=\beta_0,\quad \xi_2-\xi_1=2\beta_0. \tag{6}$$

Considering the zonal character of the flow, we obtain the following equations describing the heat transfer [2]:

$$\lambda\frac{d^2T_1}{d\xi^2}-\left[\tau_0^{\frac{1}{n}}+\left(-\mu_p\frac{dV_1}{dy}\right)^{\frac{1}{m}}\right]^n\frac{dV_1}{dy}=0,\quad 0\leqslant y\leqslant y_1,$$

$$\lambda\frac{d^2T_2}{dy^2}+\left[\tau_0^{\frac{1}{n}}+\left(\mu_p\frac{dV_2}{dy}\right)^{\frac{1}{m}}\right]^n\frac{dV_2}{dy}=0,\quad y_2\leqslant y\leqslant h,$$

$$\lambda\frac{d^2T_3}{dy^2}=0,\quad y_1\leqslant y\leqslant y_2.$$

Introducing the dimensionless temperature $\Theta = (T - T^*)/T^*$, we arrive at the dimensionless statement of the problem:

$$\begin{gathered}\frac{d^2\Theta_1}{d\xi^2}+\frac{\varkappa}{\alpha}(\xi_0-\xi)\,[(\xi_0-\xi)^{\frac{1}{n}}-\beta_0^{\frac{1}{n}}]^m=0,\quad 0\leqslant\xi\leqslant\xi_1,\\ \frac{d^2\Theta_2}{d\xi^2}+\frac{\varkappa}{\alpha}(\xi-\xi_0)\,[(\xi-\xi_0)^{\frac{1}{n}}-\beta_0^{\frac{1}{n}}]^m=0,\quad \xi_2\leqslant\xi\leqslant 1,\\ \frac{d^2\Theta_3}{d\xi^2}=0,\quad \xi_1\leqslant\xi\leqslant\xi_2\end{gathered} \tag{7}$$

with the boundary conditions (we consider only case I for the time being):

$$\Theta_1(0)=0,\quad \left.\frac{d\Theta_2}{d\xi}\right|_{\xi=1}=0 \tag{8}$$

and the conditions for the conjunction of solutions with respect to the zones

$$\begin{gathered}\Theta_1(\xi_1)=\Theta_3(\xi_1),\quad \Theta_2(\xi_2)=\Theta_3(\xi_2),\\ \left.\frac{d\Theta_1}{d\xi}\right|_{\xi_1}=\left.\frac{d\Theta_3}{d\xi}\right|_{\xi_1},\quad \left.\frac{d\Theta_2}{d\xi}\right|_{\xi_2}=\left.\frac{d\Theta_3}{d\xi}\right|_{\xi_2}.\end{gathered} \tag{9}$$

The conditions for the conjunction of solutions (9) are formulated on the basis of the additional assumption that the thermal conductivity coefficient λ does not change in passage through the boundary between the zones of the shear and the quasisolid flows, i.e.,

$$\lambda_1=\lambda_2=\lambda_3.$$

After a number of transformations, we obtain the solution of problems (7)-(9) in the following form.

For the conditions where the core is inside the flow,

$$\Theta(\xi)=\begin{cases}\dfrac{\varkappa}{\alpha}\displaystyle\sum_{k=0}^{\infty}(-1)^kF_{mn}^{k}\left\{-(\xi_0-\xi)^{\varphi_k}+\frac{\xi}{\varphi_k}\,[(1-\xi_0)^{\varphi_k-1}-2\beta_0^{\varphi_k-1}]+\xi_0^{\varphi_k},\right. & 0\leqslant\xi\leqslant\xi_1,\\ \dfrac{\varkappa}{\alpha}\displaystyle\sum_{k=0}^{\infty}(-1)^kF_{mn}^{k}\left\{\frac{\xi}{\varphi_k}\,[(1-\xi_0)^{\varphi_k-1}-\beta_0^{\varphi_k-1}]+\xi_0^{\varphi_k}-\frac{\xi_0-\beta_0}{\varphi_k}\,\beta_0^{\varphi_k-1}-\beta_0^{\varphi_k}\right\}, & \xi_1\leqslant\xi\leqslant\xi_2,\\ \dfrac{\varkappa}{\alpha}\displaystyle\sum_{k=0}^{\infty}(-1)^kF_{mn}^{k}\left\{-(\xi-\xi_0)^{\varphi_k}+\frac{\xi}{\varphi_k}\,(1-\xi_0)^{\varphi_k-1}+\xi_0^{\varphi_k}-\frac{2\xi_0}{\varphi_k}\,\beta_0^{\varphi_k-1}\right\}, & \xi_2\leqslant\xi\leqslant 1,\end{cases} \tag{10}$$

where

$$F_{mn}^{k}=C_{m}^{k}\frac{n^{2}}{(m+2n-k)(m+3n-k)};\quad \varphi_{k}=\frac{m+3n-k}{n}. \tag{11}$$

For the conditions where the core is adjacent to the upper plate,

$$\Theta_{\mathrm{up}}(\xi)=\begin{cases}\dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^{k}F_{mn}^{k}\left[-(\xi_{0}-\xi)^{\varphi_{k}}-\dfrac{1}{\varphi_{k}}\beta_{0}^{\varphi_{k}-1}\xi+\xi_{0}^{\varphi_{k}}\right], & 0\leqslant\xi\leqslant\xi_{1},\\ \dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^{k}F_{mn}^{k}\left[-\beta_{0}^{\varphi_{k}}-\dfrac{\xi_{0}-\beta_{0}}{\varphi_{k}}\beta_{0}^{\varphi_{k}-1}+\xi_{0}^{\varphi_{k}}\right], & \xi_{1}\leqslant\xi\leqslant 1.\end{cases} \tag{12}$$

For the case where the core is adjacent to the lower plate,

$$\Theta_{l_{\mathrm{p}}}(\xi)=\begin{cases}\dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^{k}F_{mn}^{k}\dfrac{1}{\varphi_{k}}\left[(1-\xi_{0})^{\varphi_{k}}-\beta_{0}^{\varphi_{k}-1}\right], & 0\leqslant\xi\leqslant\xi_{2},\\ \dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^{k}F_{mn}^{k}\left[-(\xi-\xi_{0})^{\varphi_{k}}+\dfrac{\xi}{\varphi_{k}}(1-\xi_{0})^{\varphi_{k}-1}+\beta_{0}^{\varphi_{k}}-\dfrac{\xi_{0}+\beta_{0}}{\varphi_{k}}\beta_{0}^{\varphi_{k}-1}\right], & \xi_{2}\leqslant\xi\leqslant 1.\end{cases} \tag{13}$$

For the situation where the core has "passed" beyond the upper plate, $0\leqslant\xi\leqslant 1$:

$$\Theta_{\mathrm{u}}(\xi)=\frac{\varkappa}{\alpha}\sum_{k=0}^{\infty}(-1)^{k}F_{mn}^{k}\left[-(\xi_{0}-\xi)^{\varphi_{k}}-\frac{\xi}{\varphi_{k}}(\xi_{0}-1)^{\varphi_{k}-1}+\xi_{0}^{\varphi_{k}}\right]. \tag{14}$$

For the case where the core has "passed" beyond the lower plate, $0\leqslant\xi\leqslant 1$:

$$\Theta_{l}(\xi)=\frac{\varkappa}{\alpha}\sum_{k=0}^{\infty}(-1)^{k}F_{mn}^{k}\left[-(\xi-\xi_{0})^{\varphi_{k}}+\frac{\xi}{\varphi_{k}}(1-\xi_{0})^{\varphi_{k}-1}+(-\xi_{0})^{\varphi_{k}}\right]. \tag{15}$$

Similarly, we obtain the solution for the case where the upper plate is maintained at a constant temperature, while the lower plate is insulated adiabatically. We denote this solution by $\vartheta(\xi)$. Using the same notation of the temperature profiles in relation to the flow conditions as in (10)-(15), we obtain

$$\vartheta(\xi)=\begin{cases}\dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^{k}F_{mn}^{k}\left\{-(\xi_{0}-\xi)^{\varphi_{k}}-\dfrac{\xi}{\varphi_{k}}\xi_{0}^{\varphi_{k}-1}+\dfrac{\xi_{0}^{\varphi_{k}-1}}{\varphi_{k}}+(1-\xi_{0})^{\varphi_{k}}-\dfrac{2\beta_{0}^{\varphi_{k}-1}}{\varphi_{k}}(1-\xi_{0})\right\}, & 0\leqslant\xi\leqslant\xi_{1},\\ \dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^{k}F_{mn}^{k}\left\{\xi\left[\dfrac{1}{\varphi_{k}}(\beta_{0}^{\varphi_{k}-1}-\xi_{0}^{\varphi_{k}-1})\right]+\dfrac{\xi_{0}^{\varphi_{k}-1}}{\varphi_{k}}+(1-\xi_{0})^{\varphi_{k}}-\dfrac{2\beta_{0}^{\varphi_{k}-1}}{\varphi_{k}}+\beta_{0}^{\varphi_{k}-1}\dfrac{\xi_{0}+\beta_{0}}{\varphi_{k}}-\beta_{0}^{\varphi_{k}}\right\}, & \xi_{1}\leqslant\xi\leqslant\xi_{2},\\ \dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^{k}F_{mn}^{k}\left\{-(\xi-\xi_{0})^{\varphi_{k}}+\xi\left[\dfrac{1}{\varphi_{k}}(2\beta_{0}^{\varphi_{k}-1}-\xi_{0}^{\varphi_{k}-1})\right]+(1-\xi_{0})^{\varphi_{k}}+\dfrac{1}{\varphi_{k}}\xi_{0}^{\varphi_{k}-1}-\dfrac{2}{\varphi_{k}}\beta_{0}^{\varphi_{k}-1}\right\}, & \xi_{2}\leqslant\xi\leqslant 1,\end{cases} \tag{16}$$

$$\vartheta_{\mathrm{up}}(\xi)=\begin{cases}\dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^{k}F_{mn}^{k}\left\{-(\xi_{0}-\xi)^{\varphi_{k}}-\dfrac{\xi}{\varphi_{k}}\xi_{0}^{\varphi_{k}-1}+\dfrac{\xi_{0}-\beta_{0}}{\varphi_{k}}\beta_{0}^{\varphi_{k}-1}+\beta_{0}^{\varphi_{k}}\right\}, & 0\leqslant\xi\leqslant\xi_{1},\\ \dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^{k}F_{mn}^{k}\left(\beta_{0}^{\varphi_{k}}-\dfrac{1}{\varphi_{k}}\xi_{0}^{\varphi_{k}-1}\right), & \xi_{1}\leqslant\xi\leqslant 1,\end{cases} \tag{17}$$

$$\vartheta_{l_{\mathrm{p}}}(\xi)=\begin{cases}\dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^{k}F_{mn}^{k}\left\{\dfrac{1}{\varphi_{k}}\beta_{0}^{\varphi_{k}-1}(\xi_{0}+\beta_{0})+(1-\xi_{0})^{\varphi_{k}}-\dfrac{\beta_{0}^{\varphi_{k}-1}}{\varphi_{k}}-\beta_{0}^{\varphi_{k}}\right\}, & 0\leqslant\xi\leqslant\xi_{2},\\ \dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^{k}F_{mn}^{k}\left\{-(\xi-\xi_{0})^{\varphi_{k}}+\dfrac{\beta_{0}^{\varphi_{k}-1}}{\varphi_{k}}\xi+\beta_{0}^{\varphi_{k}}-\dfrac{\beta_{0}^{\varphi_{k}-1}}{\varphi_{k}}\right\}, & \xi_{2}\leqslant\xi\leqslant 1,\end{cases} \tag{18}$$

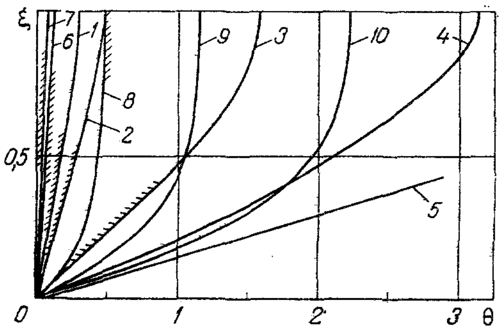

Fig. 1. Temperature profiles ($\beta_0 = 0.2$; $B = 10$): 1) $\alpha = 0.05$; 2) 0.1; 3) 0.3; 4) 0.5; 5) 0.7; 6) 0.05; 7) $\alpha = -0.1$; 8) $\alpha = -0.3$; 9) $\alpha = -0.5$; 10) $\alpha = -0.7$.

$$\vartheta_u(\xi) = \frac{\varkappa}{\alpha} \sum_{k=0}^{\infty} (-1)^k F_{mn}^k \left\{ -(\xi_0 - \xi)^{\varphi_k} - \frac{\xi_0^{\varphi_k - 1}}{\varphi_k} \xi + \frac{\xi_0^{\varphi_k - 1}}{\varphi_k} + (\xi_0 - 1)^{\varphi_k} \right\}, \quad 0 \leqslant \xi \leqslant 1, \tag{19}$$

$$\vartheta_l(\xi) = \frac{\varkappa}{\alpha} \sum_{k=0}^{\infty} (-1)^k F_{mn}^k \left\{ -(\xi - \xi_0)^{\varphi_k} + \frac{(-\xi_0)^{\varphi_k - 1}}{\varphi_k} \xi - \frac{(-\xi_0)^{\varphi_k - 1}}{\varphi_k} + (1 - \xi_0)^{\varphi_k} \right\}, \quad 0 \leqslant \xi \leqslant 1. \tag{20}$$

Numerical calculations show that the following equations are satisfied for the temperature profiles (ξ) and (ξ):

$$\Theta(\xi;\ \alpha,\ \beta_0) = \vartheta(1 - \xi;\ -\alpha,\ +\beta_0),$$

$$\frac{d}{d\xi} \Theta(1;\ \alpha,\ \beta_0) = -\frac{d}{d\xi} \vartheta(0;\ \alpha,\ \beta_0). \tag{21}$$

Considering relationships (21), we shall discuss the results only for the case of the adiabatic upper and the isothermal lower plates. A diagrammatic interpretation of the results is given for the Shvedov–Bingham model.

The temperature distributions as functions of specific values of the parameters α and β_0 are shown in Fig. 1. The hatched sections of the curves pertain to the quasisolid core.

The flow conditions are determined by the pair of parameter values (α, β_0), as was shown in [1]. This paper also provides the equations determining the type of region in the $0\alpha\beta_0$ plane corresponding to a particular set of conditions. We shall denote the region of flow with the core in the channel by D_1, the region with the core at the lower plate by D_2, the region with the core at the upper plate by D_3, and the regions where the core has "passed" beyond the lower or the upper plate by D_4 or D_5, respectively.

The mean velocity $\overline{W} = \int_0^1 W(\xi)d\xi$ for the flow conditions where the core is inside the channel depends on both the parameters α and β_0. In all other cases (even where the core is present in the flow region and is adjacent to one of the plates), the mean velocity $\overline{W}$ is determined only by the parameter α and is independent of β_0, i.e., of the core width. For instance, for (α, β_0) from D_3 [1], $\xi_0 = \beta_0 + \sqrt{-2\alpha}$, so that, with an allowance for (6) and $\alpha < 0$,

$$\overline{W} = \frac{1}{\alpha} \left[\int_0^{\xi_1} \left(\frac{\xi^2}{2} - \xi_1 \xi \right) d\xi - \frac{\xi_1^2}{2} \int_{\xi_1}^{1} d\xi \right] = \frac{1}{\alpha} \left(\frac{1}{6} \sqrt{(-2\alpha)^3} + \alpha \right).$$

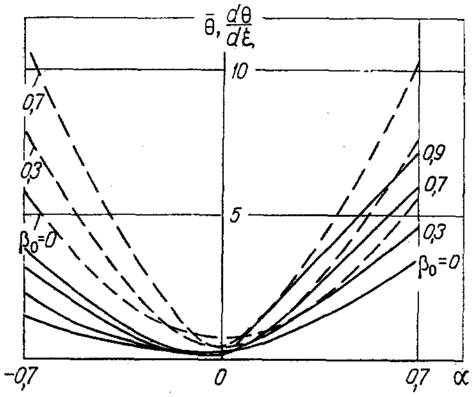

Fig. 1

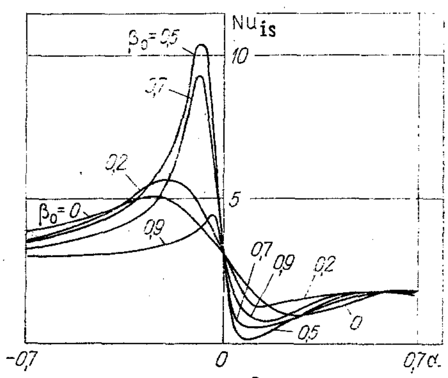

Fig. 2

Fig. 2. Distributions of the mean-mass temperature $\bar{\Theta}$ (solid curves) and of the temperature gradient $d\Theta/d\xi$ at the isothermal plate (dashed curves); B = 10.

Fig. 3. Nu numbers at the isothermal plate.

Rheodynamic conditions with an immobile core relative to the plate can be considered as pure nonplastic shear flow in a narrower channel. This can probably explain the fact that the mean velocity $\overline{W}$ is independent of the parameter β_0 that characterizes the core width.

Let us determine the relationship between the Nusselt numbers at the isothermal plate and the parameters (α, β_0, $\varkappa$),

$$\mathrm{Nu_{is}} = \frac{1}{\bar{\Theta}} \left(\frac{d\Theta}{d\xi} \right)_{\xi=0}, \tag{22}$$

where the mean-mass temperature is defined by the equation

$$\bar{\Theta} = \int_0^1 \Theta(\xi) W(\xi)\, d\xi / \int_0^1 W(\xi)\, d\xi.$$

Considering that the qualities figuring in (22) are given by

$$\Theta(\xi) = \frac{\varkappa}{\alpha} \Phi^T_{mn}(\xi;\ \alpha,\ \beta_0), \quad W(\xi) = \frac{1}{\alpha} \Phi^{\mathrm{ve}}_{mn}(\xi;\ \alpha,\ \beta_0),$$

$$\left.\frac{d\Theta}{d\xi}\right|_{\xi=0} = \frac{\varkappa}{\alpha} \Phi^{\mathrm{bo}}_{mn}(\alpha,\ \beta_0),$$

we obtain

$$\mathrm{Nu_{is}} = \frac{\dfrac{\varkappa}{\alpha} \Phi^{\mathrm{bo}}_{mn}(\alpha,\ \beta_0) \dfrac{1}{\alpha} \displaystyle\int_0^1 \Phi^{\mathrm{ve}}_{mn}(\xi;\ \alpha,\ \beta_0)\, d\xi}{\dfrac{\varkappa}{\alpha} \dfrac{1}{\alpha} \displaystyle\int_0^1 \Phi^T_{mn}(\xi;\ \alpha,\ \beta_0)\, \Phi^{\mathrm{ve}}_{mn}(\xi;\ \alpha,\ \beta_0)\, d\xi} = \psi_{mn}(\alpha,\ \beta_0). \tag{23}$$

Consequently, for the problem under consideration, the intensity of heat exchange at the isothermal plate does not depend on the dissipative parameter

$$B = \frac{\varkappa}{\alpha} = \mathrm{Br\,Sen} \frac{1}{\alpha\beta_0} = \frac{Ah^3 (Ah)^{\frac{m}{n}}}{\lambda \mu_p T^*};$$

it is determined only by the pair of (α, β_0) values for the assigned values of m and n.

The functions $\psi_{mn}(\alpha,\ \beta_0)$ are very cumbersome, however, in principle, it is difficult to derive them from the obtained solutions for each specific set of flow conditions.

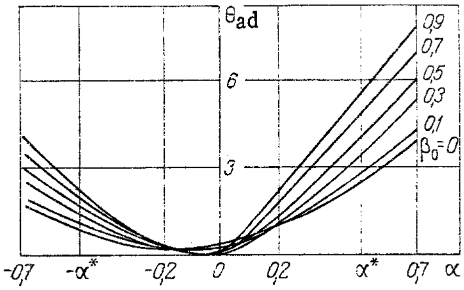

Fig. 4. Heating of the adiabatic plate as a function of the parameters α and β_0; B = 10.

The role of the rheological factor in heat exchange at the isothermal plate is elucidated in Figs. 2 and 3. The maximum heat exchange intensity occurs when the vectors **A** and **U** are in opposition. According to calculations, the highest heat exchange intensity occurs for $|\alpha| \sim 0.08$ and $\beta_0 \sim 0.5$, i.e., for $A \sim 2\tau_0/h$ and $U \sim 0.16\tau_0 h/\mu_p$. The minimum heat exchange intensity is observed for $A \sim 2\tau_0/h$ and $U \sim 0.1\tau_0 h/\mu_p$ with the vectors **A** and **U** having the same sense.

We readily obtain the expressions for the heating of the adiabatically insulated plate:

$$\Theta_{ad} = \begin{cases} \dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^k F_{mn}^k\left[-(1-\xi_0)^{\varphi_k}+\dfrac{(1-\xi_0)^{\varphi_k-1}}{\varphi_k}+\right. \\ \qquad\left.+\,\xi_0^{\varphi_k}-\dfrac{2\xi_0\beta_0^{\varphi_k-1}}{\varphi_k}\right], \ D_1, \\ \dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^k F_{mn}^k\left[-(1-\xi_0)^{\varphi_k}+\dfrac{(1-\xi_0)^{\varphi_k-1}}{\varphi_k}+\right. \\ \qquad\left.+\,\beta_0^{\varphi_k}-\dfrac{\xi_0+\beta_0}{\varphi_k}\beta_0^{\varphi_k-1}\right], \ D_2, \\ \dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^k F_{mn}^k\left[-\beta_0^{\varphi_k}-\dfrac{\xi_0-\beta_0}{\varphi_k}\beta_0^{\varphi_k-1}+\xi_0^{\varphi_k}\right], \ D_3, \\ \dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^k F_{mn}^k\left[-(1-\xi_0)^{\varphi_k}+\dfrac{(1-\xi_0)^{\varphi_k-1}}{\varphi_k}+(-\xi_0)^{\varphi_k}\right], \ D_4, \\ \dfrac{\varkappa}{\alpha}\sum\limits_{k=0}^{\infty}(-1)^k F_{mn}^k\left[-(\xi_0-1)^{\varphi_k}-\dfrac{1}{\varphi_k}(\xi_0-1)^{\varphi_k-1}+\xi_0^{\varphi_k}\right], \ D_5. \end{cases} \tag{24}$$

The curves of Θ_{ad} as functions of the parameters α and β_0 for the fixed value B = 10 are shown in Fig. 4. It is evident that, for a fixed β_0 (i.e., for the assigned value of the pressure gradient A), the heating of the adiabatic plate increases with $|\alpha|$ (i.e., with an increase in the velocity of the upper plate U). The heating is more intensive if the vectors **A** and **U** have the same sense. It should also be noted that, for fixed β_0 values and $|\alpha| > |\alpha^*(\beta_0)|$, the relationship $\Theta_{ad}(\alpha)$ becomes linear. Here, $\alpha^*(\beta_0)$ is the value corresponding to the "departure" of the core from the flow region.

The relationships (16)-(20) and (21) can be used in a similar manner to obtain the results for the intensity of heat exchange at the isothermal plate and the heating of the adiabatic plate in case II.

NOTATION

Dimensional quantities: λ, thermal conductivity coefficient; U, velocity of the upper plate; grad p = A, pressure gradient; τ, shearing stress; τ_0, ultimate shearing stress; μ_p, analog of plastic viscosity; m and n, nonlinearity parameters of the flow curve; h, channel width; y, vertical coordinate; y_1 and y_2, core boundaries; V(y), flow velocity; T(y), temperature of the medium; $\dot{\gamma} = dV/dy$, shearing rate; T^*, temperature of the isothermal plate. Dimensionless quantities: $W = V/U$, flow velocity; $\Theta = (T - T_l^*)/T_l^*$; $\vartheta = (T - T_u^*)/T_u^*$, temperature of the medium; $\xi = y/h$, vertical coordinate; ξ_1 and ξ_2, core boundaries; ξ_0, coordinate of the plane where the shearing stress vanishes; $\alpha = \mu_p U/(Ah)^{m/n}h$, $\beta_0 = \tau_0/Ah$, and $\varkappa = AUh^2/\lambda T^*$, parameters; $B = \varkappa/\alpha$, dissipative parameter; Nu_{is}, Nusselt numbers at the isothermal plate; $Sen = \tau_0 h/\mu_p U$, St. Venant–Il'yushin number; $Br = \mu_p U^2/\lambda T^*$, Brinkman number.

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EFFECT OF SELF-OSCILLATIONS ON THE HYDRAULIC RESISTANCE OF A VORTEX TUBE

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UDC 532.526.4

It has been established experimentally that the hydraulic loss increases as tangential self-oscillations develop in a swirled flow.

Swirling of liquid and gas flows is widely used in modern technology as an effective means of intensifying the heat and mass exchange processes. It has been noted that the expenditure of energy on flow advancement increases with an increase in the heat exchange intensity. This effect is most strongly pronounced in the case of short vortex tubes with high vorticity at the inlet [1]. The increase in the heat exchange intensity and the resistance is usually explained by the effect of mass forces, which generate secondary flows and an elevated turbulence level. The other well-known characteristic of a swirled flow – its capacity for spontaneous excitation of intensive, regular velocity and pressure pulsations – is usually not taken into account. However, the results of many experiments indicate that self-oscillations are closely related to transport processes [2]. Thus, the highest energy exchange intensity and a considerable reduction in the throughput of a vortex tube are observed under conditions where the pulsation amplitude is at a maximum. The present article advances the hypothesis that self-oscillation processes are among the most important factors which determine the acceleration of heat exchange and the increase in the hydraulic resistance in a vortex tube. The experimental data given below refer to the interrelationship between oscillations and the hydraulic loss, and they support to a certain extent the above point of view.

The experimental simulator is shown schematically in Fig. 1. An endless-screw swirler 2 is mounted in a cylindrical tube 1, whose length is L and the radius r_0 = 8 mm. The dimensions of 10 different screws make it possible to vary the degree of flow vorticity in the range from A = 1.76 to A = 16. The vorticity parameter is calculated by means of the expression $A = \pi r_0^2 \sin \beta/F_{in}$ after Abramovich [3]. The swirler is fastened on a mobile hollow rod 3; by moving this rod, the distance L can be varied in the 5-400-mm range. If necessary, the central cavity of the tube can be made to communicate with the ambient by opening the valve 4 inside the through-passage in the rod. The sides of the tube are provided with holes 5 which allow a dye to be fed into the flow to make it visible or allow pressure and velocity data units to be mounted.

The water flow 6, which arrives from the delivery branch pipe 7 while the valve 4 is in the "closed" position, produces in the cylindrical tube beyond the swirler a hollow vortex 8, which performs combined rotary and translational motions. Liquid from the ambient is drawn

Kuibyshev Aviation Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 7, pp. 59-64, July, 1979. Original article submitted June 2, 1978.

0022-0841/79/3701-0796$07.50